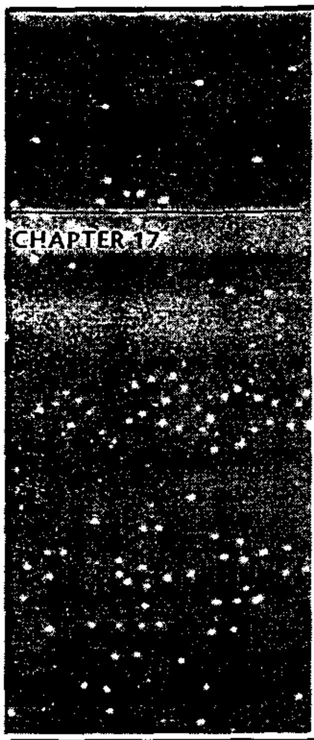


# Comparisons of Spatial Approximations

VERY SYSTEMATIC ANALYSES can be carried out comparing some nodal methods, such as the Analytic Nodal Method, with mesh centered finite differences methods. A good comparison can be made using a quite realistic CANDU-6 reactor model. This comparison is the subject of this chapter. It is an investigation that was carried out not too long ago, and is in the process of being published in a scientific journal (Accepted for publication, October 1997, *Annals of Nuclear Energy*, by Jean Koclas).

## Analytic Nodal Method

The Analytic Nodal Method was derived at MIT in the late seventies. It is derived following exactly the same steps we have used in chapter 15, *Mesh Centered Finite Differences*, page 161. However, it does not truncate the exponential matrices, using exact expressions for the matrix exponentials that appear. It also assumes a quadratic approximation to the transverse leakage terms, by fitting the average transverse leakages to a quadratic polynomial.The resulting equations are then solved analytically. Therefore, the mesh centered finite differences are the lowest order approximation to this nodal method.

It is of interest to compare the results of a CANDU reactor calculation performed with the Analytic Nodal Method, and with the Mesh Centered Finite Differences. This provides an independent verification that the methods in use for reactor analysis, and the mesh spacings as well, are adequate for their purposes. The calculations are all performed for the reactor in the steady state.

## Reactor Model

We use in this analysis the reactor model described in chapter 6, *Spatial Mesh Considerations*, page 49. However, there is the presence of 21 adjuster rods, and 14 zone controllers that will have a strong flux flattening effect. Structural materials are absent from the model used, and Xenon-135 distribution in the core was not taken into account. The axial notch in the reflector thickness was not modeled either, to simplify the geometry description in the Analytic Nodal Model computer code.

## Comparisons

We use the Analytic Nodal Method with the quadratic leakage approximation as a reference. The effect of two lower leakage approximations are examined, one with a flat or constant leakage across the node interfaces, and one with zero leakage.

Then variations on the coarse mesh finite difference methods are examined. We start with a quadratic leakage approximation, then a flat leakage, and finally a zero leakage (the standard method) approximation.

## Effective Multiplication Factors

As a first measure of error, we summarize in a table the $K_{eff}$ of the various approximations.

**TABLE 3.** Effective Multiplication Factor Comparisons

| Method | Leakage Approximation | $K_{eff}$ | % Difference |
|---|---|---|---|
| ANM | Quadratic | 1,0300977 | — |
| ANM | Flat | 1,0302858 | 0,018260 |
| ANM | 0 | 1,0299739 | -0,012018 |
| MCFD | Quadratic | 1,0304341 | 0,032657 |
| MCFD | Flat | 1,0306158 | 0,050296 |
| MCFD | 0 | 1,0302292 | 0,012766 |

It is obvious from this table that the Analytic Nodal Method, with any of the spatial approximations, is superior to the Mesh Centered Finite Differences. This is a good indication that the intra node flux shape plays an important role in the accuracy of the method. It also appears that neglecting entirely the transverse leakages in Mesh Centered Finite Differences gives better multiplication factors than when the leakages are present. This may be due to some kind of error cancellation.

However, it is also a fact that the $K_{eff}$ is not the only performance indicator available. The errors on the fluxes is also very important, and probably the most important factor, because many safety related parameters are related to the fluxes.

## Errors on the Fluxes

In order to estimate the errors on the flux distributions, the reactor core was divided in nine very coarse regions, over which the fluxes were averaged. This is shown on Figure 11, "Coarse Regions for Flux Averages", page 202.

The average fluxes for each of the studied methods appear on Figure 12, "Flux Distribution", page 203. The corresponding errors, using the Analytic Nodal Method as as reference, are shown on Figure 13, "Errors on the Fluxes", page 204.

Once again, we can infer that any of the Analytic Nodal Method approximations are superior to any of the Mesh Centered Finite Differences. Is is also quite clear that quadratic leakage approximations deteriorate the performance of the Mesh Centered Finite Differences. Overall, it seems that a flat transverse leakage approximation tends to improve the flux distribution, albeit marginally. The error histograms of the full 3-D fluxes are shown on Figure 14, "Error Histogram for ANM-Flat Transverse Leakages", page 205, to Figure 18, "Error Histogram for CMFD-0 Transverse Leakages", page 209, and they also help confirm this conclusion.

The question is still an open one. In statics calculations, is it preferable to increase the number of unknown leakages with a flat leakage term, or is it better to use a finer mesh with zero leakages? Of course, if space time kinetics calculations are to follow, it seems that the extra unknowns of the flat leakage approximation are "better" than the extra mesh points, because they do not involve extra delayed precursor unknowns to carry along in the computation. Some work is still needed to elucidate all of this.

FIGURE 11. **Coarse Regions for Flux Averages**

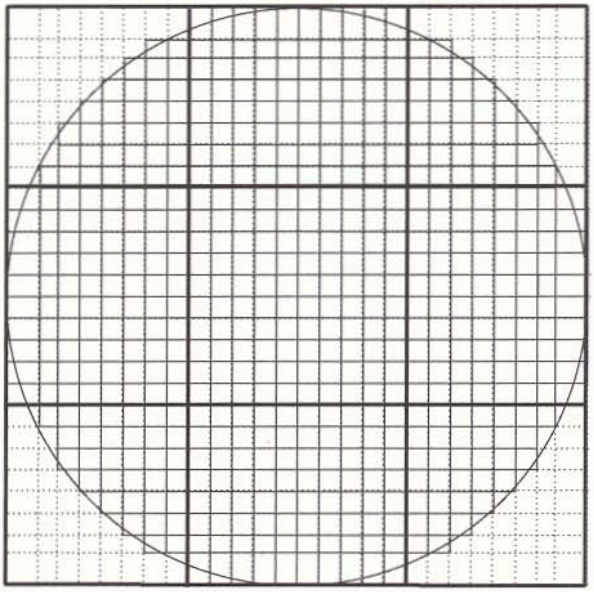

**FIGURE 12. Flux Distribution**

| | | | |
|---|---|---|---|
| Ref | 0,743509 | 0,967568 | 0,744399 |
| ANM-F | 0,744600 | 0,969243 | 0,745570 |
| ANM-0 | 0,747004 | 0,965676 | 0,746998 |
| CMFD-Q | 0,736493 | 0,973215 | 0,736489 |
| CMFD-F | 0,737266 | 0,974988 | 0,738937 |
| CMFD | 0,736629 | 0,971168 | 0,736626 |
| | 0,988337 | 1,14049 | 0,988293 |
| | 0,989860 | 1,13590 | 0,990495 |
| | 0,988709 | 1,13909 | 0,988691 |
| | 0,987087 | 1,14444 | 0,987071 |
| | 0,988546 | 1,13988 | 0,989366 |
| | 0,986976 | 1,14546 | 0,986960 |
| | 0,809804 | 1,02840 | 0,809785 |
| | 0,809631 | 1,02994 | 0,811746 |
| | 0,814522 | 1,02790 | 0,814508 |
| | 0,798085 | 1,02753 | 0,798073 |
| | 0,797568 | 1,02903 | 0,799771 |
| | 0,799525 | 1,02719 | 0,799513 |

## FIGURE 13. Errors on the Fluxes

| | | | |
|---|---|---|---|
| Ref | — | — | — |
| ANM-F | 0,1467 | 0,1731 | 0,1563 |
| ANM-0 | 0,4701 | -0,1955 | -0,3491 |
| CMFD-Q | -0,9436 | 0,5836 | -1,0626 |
| CMFD-F | -0,8397 | 0,7664 | -0,7337 |
| CMFD | -0,9253 | 0,3721 | -1,0442 |
| | — | — | — |
| | 0,1541 | -0.4025 | -0,2228 |
| | 0,0376 | -0,1228 | -0,0403 |
| | 0,1265 | 0,3463 | -0,1236 |
| | 0,0211 | -0,0535 | 0,1086 |
| | 0,1377 | 0,4358 | -0,1349 |
| | — | — | — |
| | -0,0214 | 0,1497 | 0,2422 |
| | 0,5826 | -0,0486 | 0,5832 |
| | -1.4471 | -0,0846 | -1,4463 |
| | -1.5110 | 0,0613 | -1,2366 |
| | -1.2693 | -0,1177 | -1,2685 |

FIGURE 14. **Error Histogram for ANM-Flat Transverse Leakages**

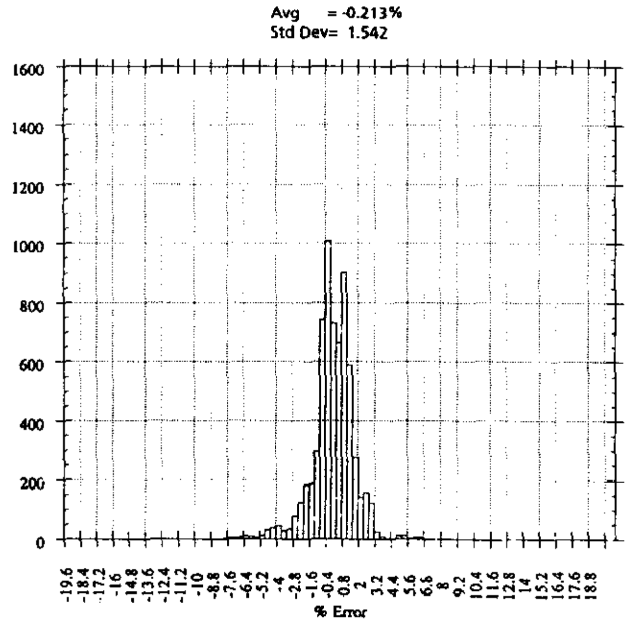

FIGURE 15. Error Histogram for ANM-Zero Transverse Leakages

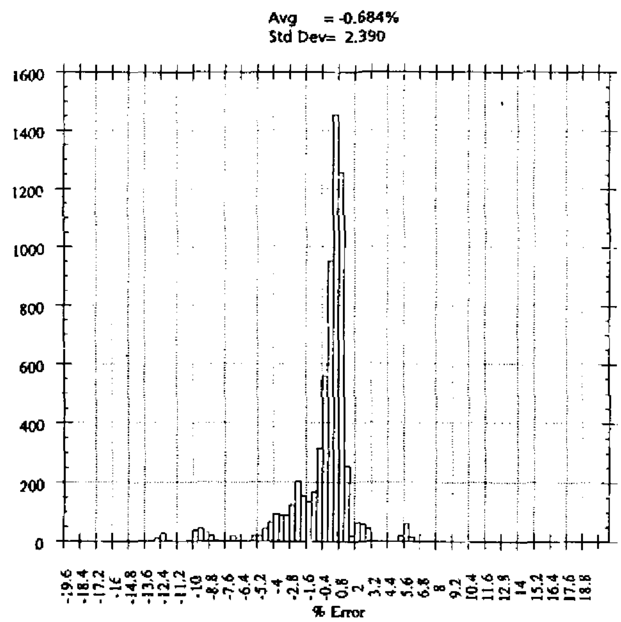

FIGURE 16. **Error Histogram for CMFD-Quadratic Transverse Leakages**

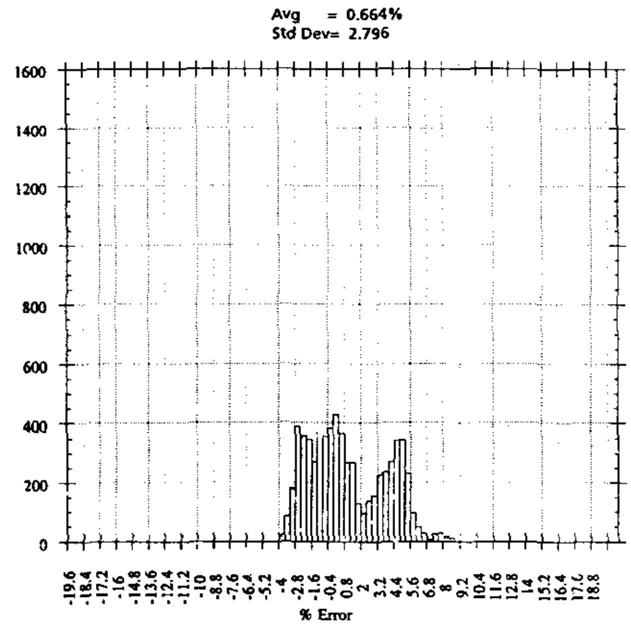

FIGURE 17. Error Histogram for CMFD-Flat Transverse Leakages

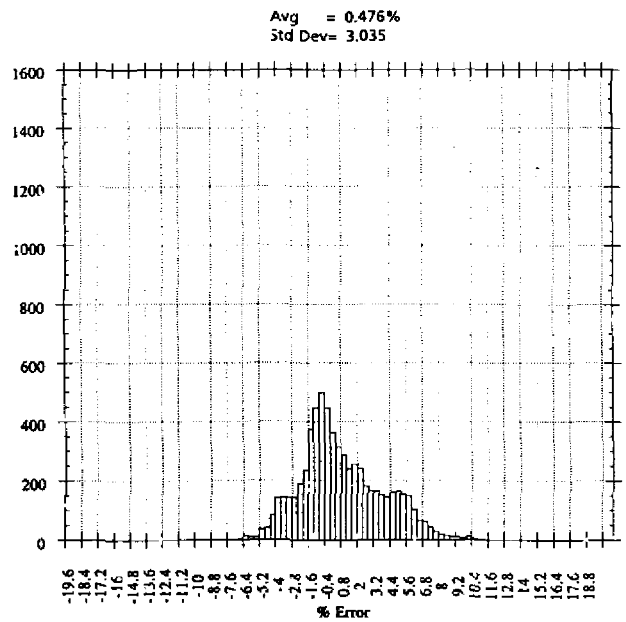

FIGURE 18. Error Histogram for CMFD-0 Transverse Leakages